

Increase in the Percentage versus Percentage Increase versus Percentage Reduction in Error

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This memo concerns knowledge, measured at the end of an experiment whose treatment is designed to increase it. Following Prior and Lupia, I consider the post-treatment (t_2) contrast between the treatment and control groups, although the same points hold if the contrast is instead between the treatment group at t_2 versus t_1 . I shall speak loosely of “increases,” meaning, strictly speaking, in the Prior-Lupia context, upward differences.

Denote the control group’s mean t_2 knowledge level by K_1 , and the treatment group’s by K_2 . To simplify the formulas and the arithmetic, let us take these to be proportions, while continuing (for sake of consistency with the text) to refer to them as percentages (the proportions times 100). Then the increase in the percentage is

$$\Delta_K = K_2 - K_1;$$

the percentage increase is

$$\Delta_K^{\%} = (K_2 - K_1) / K_1;$$

and the percentage reduction in error is

$$\Delta_{(1-K)}^{\%} = [(1 - K_1) - (1 - K_2)] / (1 - K_1) = (K_2 - K_1) / (1 - K_1).$$

Thus, for example, $K_1 = .60$ and $K_2 = .80$ implies $\Delta_K = .80 - .60 = .20$, $\Delta_K^{\%} = (.80 - .60) / .60 = .333\dots$, and $\Delta_{(1-K)}^{\%} = (.60 / .40)(.333\dots) = .50$.

The most important point to note about the difference in the percentage Δ_K is that it is the treatment effect. As anyone reading this probably knows, the standard definition of a regressor’s, say x_1 ’s, effect (needing some footnoting in the presence of endogeneity) is as the partial derivative of the conditional expectation of the dependent variable with respect to x_1 : $\partial(E(y_i | X_i) / \partial x_{i1})$, where X_i is the vector of the i th observation on all the equation’s regressors. In a linear model that makes x_1 ’s effect γ_1 , its coefficient. In the present context, where there’s just one, binary regressor, scored $\{0,1\}$, γ_1 works out in turn to the difference of proportions, Δ_K .

The percentage difference $\Delta_K^{\%}$ and the percentage reduction in error $\Delta_{(1-K)}^{\%}$, by contrast, are normings, dividing the effect by K_1 and $1 - K_1$, respectively. Since $0 \leq K_1 \leq 1$, both $\Delta_K^{\%}$ and

$\Delta_{(1-K)}^{\%}$ must necessarily exceed Δ_K (in absolute value, should Δ_K be negative), except when $K_1 = 0$ (in which case, $\Delta_{(1-K)}^{\%} = \Delta_K$, and $\Delta_K^{\%}$ is undefined) or $K_1 = 1$ (in which case $\Delta_K^{\%} = \Delta_K$, and $\Delta_{(1-K)}^{\%}$ is undefined). The departure from Δ_K can be either tiny or vast. If $K_1 = .01$ and $K_2 = .02$, $\Delta_K = .01$ and $\Delta_{(1-K)}^{\%} = .0101$, but $\Delta_K^{\%} = 1.00$. On the other hand, if $K_1 = .99$ and $K_2 = 1.00$, Δ_K again = .01 and $\Delta_K^{\%}$ now = .0101, but $\Delta_{(1-K)}^{\%}$ now = 1.00. I.e., a 1% increase in knowledge is a 100% percentage increase but only a 1.01% percentage reduction in error if it is an increase from 1% to 2% and is a 100% percentage reduction in error but only a 1.01% percentage increase if it is an increase from 99% to 100%.

As these last examples suggest, $\Delta_K^{\%}$ and $\Delta_{(1-K)}^{\%}$ are intimately and inversely related, each an upside-down version of the other. More precisely,

$$\Delta_{(1-K)}^{\%} = [K_1 / (1 - K_1)] (\Delta_K^{\%}),$$

(excluding the cases in which $K_1 = 0$ or 1 and either $\Delta_K^{\%}$ and $\Delta_{(1-K)}^{\%}$ is therefore undefined). The two are equal at $K_1 = .5$. For $K_1 < .5$, $\text{Abs}(\Delta_K^{\%}) > \text{Abs}(\Delta_{(1-K)}^{\%})$; for $K_1 > .5$, the reverse. At the extremes, as we have seen, the one will be huge, the other scarcely more than Δ_K . So the percentage increase $\Delta_K^{\%}$, the formula Prior and Lupia employ, is the one that will make increases from low to lowish initial percentages ($K_1 < .5$, as in Prior and Lupia's results) look largest. Again, however, both $\Delta_K^{\%}$ and $\Delta_{(1-K)}^{\%}$ are mere forms of elaboration. The actual treatment effect is the increase (or decrease) in the percentage.